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Causal reasoning and explanation in dynamic economic systems

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In this paper, we develop techniques for qualitative reasoning in economic systems. It is shown that qualitative economic reasoning can be formalised to the extent that qualitative behaviour and associated explanations can be obtained which correspond to economic reasoning put forward by economists. Existing qualitative reasoning techniques are capable of generating all possible behaviours of an economic system out of which many have no satisfactory economic explanation. In order to constrain this intractable branching of behaviour, a heuristic filter based on the causal dependencies in the economic system selects those behaviours which are meaningful from an economic point of view.

1. Introduction

Economic theory is partially concerned with modelling complex economic systems. Economists are often interested in determining the equilibrium values of relevant variables in static models and in the characteristics of the solutions to dynamic models. Considering their evolution during the last few decades, economic models have increased in size, e.g., measured by the number of equations. About forty years ago, the average number of equations in macroeconometric models was approximately 20. Nowadays, there are models consisting of more than 500 equations [see, e.g., van den Berg, Gelauff, and Okker (1988), Burridge et al. (1991)]. Other trends leading to greater complexity in economic models are: linking together several country models into a global model and the incorporation of nonlinear equations [Waelbroeck (1976)]. This growth in complexity has also been induced by the tremendous increase in

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computer power available. Apart from the increase in computer speed, library programs for econometric routines and computerised data banks greatly facilitated economic model building.

The disadvantage of this growth is that the economic modelling process tends to become unmanageable. Often, the computer output of large models is almost intractable,[1] which complicates the interpretation of the results. Furthermore, investigators are confronted with substantial specification uncertainty when constructing econometric models for testing economic hypotheses [Blommestein (1985)]. This follows from the observation that economic theories are usually not detailed enough to warrant a one-to-one mapping from theory to fully specified models. In other words, the correspondence between equations of a structural econometric model and the underlying economic theory, from which the equations are derived, is often disturbed. In such cases, it is difficult to obtain a causal description of the results of the simulation and questions about the relevance of the results can be difficult to answer [Royer and Ritschard (1984)].

Contrary to these developments, textbooks treat simple economic models and focus mainly on qualitative aspects and explanation. The issue is to explain the effects of a change in an exogenous parameter on the other variables in a qualitative sense, i.e., without using number crunching methods. This paper deals with the formulation and also the application of qualitative techniques to a simple economic model.

The treatment of qualitative systems by Samuelson (1947) is generally considered to be the first contribution to formal qualitative systems. In the last decade, AI researchers started to develop theories of qualitative reasoning (QR) mainly to study problems in the domain of physics, electronic circuits, and medical diagnosis. At present, the assimilation of QR techniques in the economics domain has been fairly limited, although some interesting contributions have been made [Berndsen and Daniels (1990), Bourgine and Raiman (1986), Farley (1986), Karakoulas (1990), Pau and Gianotti (1990)]. In this paper, we investigate further application of QR techniques in economics. More specifically, we focus upon aspects of developing an automatic procedure to analyse and explain the possible qualitative behaviours of economic textbook models. Explanation is an important issue in economic modelling since results of a computer simulation or analysis are more easily accepted when they are accompanied by a causal explanation [see, e.g., Blommestein (1985)].

The research presented here is purely at the methodological level rather than at the domain level, i.e., the goal is not to devise a new economic theory but to develop tools for analysing the structure of qualitative economic models and to provide causal accounts of the possible distinct behaviours of such models.

[1] In Rauh (1988), the imaginative term 'Zahlenfriedhof' ('numbers graveyard') is used.

It is well-known that standard qualitative reasoning techniques suffer from a phenomenon called intractable branching. In the literature, a number of proposals have been made to tame intractable branching based on various mathematical techniques [Struss (1988)]. Also in qualitative comparative statics [Royer and Ritschard (1984)] it is shown that qualitative multipliers are almost always ambiguous. It is therefore that the application of heuristics is inevitable to obtain meaningful results. In this paper, the emphasis is on developing techniques to reduce branching in ways which are meaningful from an *economic* point of view. The causality heuristic is formulated as a discriminating rule filtering states from the simulation which do not meet the criterion of the heuristic. This heuristic reflects a common principle of economic reasoning.

The outline of the paper is as follows. In section 2, we present a standard framework suitable for representing qualitative economic systems in constraints. The system is represented as a directed graph called constraint graph. In section 3, the theoretical framework of the causality heuristic is described. This results in a procedure to obtain a causal influence graph which shows the causal dependencies of the economic system.

In section 4, we apply these techniques to the well-known Mundell–Fleming model [Fleming (1962), Mundell (1962)]. A pure Qualitative Simulation approach would lead to completely intractable results, whereas the formal application of the causality heuristic provides a transparent description of the behaviour corresponding to what is found in economic textbooks. In section 5, we briefly discuss the computer program QERT (Qualitative Economic Reasoning Tool). This program is an implementation of the techniques presented in sections 3 and 4.

2. Representation of qualitative economic systems

In this section we present a standard formalism for qualitative economic systems [Berndsen and Daniels (1991)]. An *economic system* $\mathcal{S}$ is defined as the tuple $\langle \mathcal{V}, \mathcal{Q}, \mathcal{C} \rangle$, where:

- $\mathcal{V}$ is a set of variables $\{v_1, \ldots, v_n\}$.
- $\mathcal{Q}$ is a set of quantity spaces *QSval* and *QSdir* for every variable v.
- $\mathcal{C}$ is a set of constraints.

This definition is not restricted to economic systems but in fact covers any problem which can be formulated as a finite domain constraint problem. However, in this paper we consider the type of constraints suitable for representing economic relations.

In economics, events are usually described in discrete time intervals corresponding to accounting periods, e.g., a quarter or a year. Therefore, time is represented by a finite set of m half-open time intervals of uniform length

$T = \{[t_0, t_1\rangle, \ldots, [t_{m-1}, t_m\rangle\}$ or, compactly, as $\{t_1, \ldots, t_m\}$. The function of time is to impose an ordering on the states of the system. However, the division of time into discrete intervals of equal length is not crucial from a technical point of view [compare, e.g., Kuipers (1986), Williams (1986)]. $Qval(v, t_k)$ denotes the *qualitative value* of variable v at the beginning of time interval $[t_k, t_{k+1}\rangle$. $Qdir(v, t_k)$ denotes the *qualitative direction of change* of v in time interval $[t_k, t_{k+1}\rangle$. $QSval$, $QSdir \in \mathscr{Q}$ are called *quantity spaces* and specify the range of values that a variable can take on. Various quantity spaces have been proposed [Forbus (1984), Kuipers (1986), Raiman (1986), Travé-Massuyès and Piera (1989)].

Quantity spaces contain a *finite number of symbolic values*. In our case, the quantity spaces $QSval$ and $QSdir$ are fixed over time and totally ordered. Here, $QSval = \{-, 0, +\}$, where '0' denotes the value of variable v at the beginning of the first time interval $[t_0, t_1\rangle$, '$-$' and '$+$' indicate that v is under or above its initial value. In the special case in which the initial value of v equals $0 \in \Re$, the values of $QSval$ can be mapped onto the set of real intervals $\{(-\infty, 0\rangle, 0, \langle 0, \infty\rangle\}$. In general, let the initial value of v equal $a \in \Re$, then $\{-, 0, +\}$ is associated with $\{\langle -\infty, a\rangle, a, \langle a, \infty\rangle\}$. In case the quantity space of a variable is restricted to the positive segment of the real line, $\{-, 0, +\}$ corresponds to $\{\langle 0, a\rangle, a, \langle a, \infty\rangle\}$. In some applications, the $Qval$ of a variable is not important; then we define formally, $QSval = \{?\}$, where ? is shorthand notation for $\langle -\infty, \infty\rangle$ or $\langle 0, \infty\rangle$. The quantity space $QSdir = \{dec, std, inc\}$. The interpretation of this quantity space is that v is decreasing, steady, or increasing if, respectively, $Qdir(v) = dec, std$, or inc.

A *qualitative state* $QS(v_j, t_k)$ of a variable v_j at t_k is defined as the tuple $(Qval(v_j, t_k), Qdir(v_j, t_k))$. An *admissible* qualitative state $QS(\mathscr{V}, t_k)$ is a qualitative state of $\mathscr{S}$ such that all constraints in $\mathscr{C}$ are satisfied simultaneously. The corresponding assignment of qualitative states to variables is called a *valid interpretation.*

At this point, it may be illustrative to inspect the appendix for an example of a constraint-based qualitative model. The model is a version of the well-known Mundell–Fleming model [Fleming (1962), Mundell (1962)] which serves as an example throughout this paper. The reader is invited to observe the correspondence between the constraints and the economic relations. The semantics of the constraints which may occur in $\mathscr{C}$ are defined below. The straightforward definition of the qualitative operators addition ($\oplus$), unary minus ($\ominus$), and multiplication ($\otimes$) may be found in Berndsen and Daniels (1990).

SUM-constraint

The constraint $\mathrm{SUM}(v_1, (s_2, v_2), \ldots, (s_n, v_n))$ defines v_1 as the qualitative sum $s_2 v_2 \oplus \cdots \oplus s_n v_n$, where $s_i \in \{+, -\}$ is the sign of v_i $(i = 2, \ldots, n)$.

$\mathrm{SUM}(v_1, (s_2, v_2), \ldots, (s_n, v_n))$ is satisfied at time t if

$$Qdir(v_1, t) \approx (s_2 \otimes Qdir(v_2, t)) \oplus \cdots \oplus (s_n \otimes Qdir(v_n, t)).$$

Furthermore, an analogous condition with respect to the *Qvals* must hold.

SC-constraint

The constraint $\mathrm{SC}(v_1, (s_2, v_2), \ldots, (s_n, v_n))$ defines a relation of sequential causality between v_1 and $n-1$ other variables. For each of the $n-1$ variables v_i, the sign $s_i \in \{+,-\}$ $(i = 2, \ldots, n)$ denotes the effect of one-period lagged v_i on v_1 if the ceteris paribus condition with respect to the $n-2$ other variables holds. $\mathrm{SC}(v_1, (s_2, v_2), \ldots, (s_n, v_n))$ is satisfied at time t_k if

$$Qdir(v_1, t_k) \approx (s_2 \otimes Qdir(v_2, t_{k-1})) \oplus \cdots \oplus (s_n \otimes Qdir(v_n, t_{k-1})).$$

M-constraint

The monotonicity constraints $\mathrm{M}^+(a, b)$ and $\mathrm{M}^-(a, b)$ define a monotonic functional relationship between a and b. M^+ is appropriate if the relationship between a and b is monotonic and increasing. Conversely, if the relationship is decreasing and monotonic, the M^--constraint applies. The monotonicity constraint $\mathrm{M}^+(a, b)$ is satisfied iff $QS(a) = QS(b)$; similarly, $\mathrm{M}^-(a, b)$ is satisfied iff $QS(a) = (\ominus QS(b))$. The monotonicity constraint can be employed to model relationships in which two variables are contemporaneous, i.e., the variables refer to the same time interval and have a mutual influence. Note that the M-constraint is not equivalent to a degenerate SUM-constraint. This is due to the difference between weak equality $(\approx)$ and strong equality $(=)$ of qualitative values.

DERIV-constraint

The qualitative analogue of the derivative relation between two variables a and b is represented by the constraint $\mathrm{DERIV}(a, b)$, where b is the qualitative time derivative of a. $\mathrm{DERIV}(a, b)$ is satisfied at t_k iff the pair $(Qdir(a, t_k), Qval(b, t_k))$ matches one of the entries below. The quantity space $QSval(b)$ must equal $\{-, 0, +\}$.

$\mathrm{DERIV}(a, b)$	$Qdir(a, t_k)$	$Qval(b, t_k)$
	std	0
	inc	+
	dec	−

The DERIV-constraint may be employed to describe the tâtonnement-adjustment process of a market, i.e., DERIV(P, Exd) where P is the price variable and Exd denotes excess demand. Thus, if there is excess demand (supply), the price is increasing (decreasing); if the market is in equilibrium, the price is steady.

EXO-constraint

An exogenous variable $a \in \mathscr{V}$ is denoted by the unary constraint EXO(a). Because the values of exogenous variables are determined outside the economic system, a list of qualitative states for each exogenous variable a, $QS(a, t_1), \ldots, QS(a, t_m)$, is defined which is associated with EXO(a).

An economic system can also be represented as a directed graph as follows:

Definition 1 (*constraint graph*). *A constraint graph G of $\mathscr{S}$ is a labelled directed graph with a finite vertex set V corresponding to the set of economic variables $\mathscr{V}$ and a set of directed links L; there is a directed link from v to w, denoted vw, iff v and w ($v \neq w$) appear in the same constraint $c \in \mathscr{C}$ in the correct order as defined in table 1. The label l_{vw} is associated with link vw.*

Table 1

Label set.

Constraint c	Label l_{vw} of link vw
SUM($w, \ldots, (s_v, v), \ldots$)	s_v
SC($w, \ldots, (s_v, v), \ldots$)	sc s_v
$M^+(w, v)$ or $M^+(v, w)$	+
$M^-(w, v)$ or $M^-(v, w)$	−
DERIV(w, v)	deriv

For example, the constraint graph of the Mundell–Fleming model[2] is depicted in fig. 1. The constraint EXO(v) is not included in table 1 because it does not induce a directed link in G; exogenous variables appear in G as nodes with zero in degree.

It is easy to show that the interpretation of the constraint graph, more specifically, the direction of the links in G, is analogous to the way in which the equations of an economic model are interpreted. Usually, economic equations are written in a canonical form in which the left-hand side of each equation

[2]This model can be specified in two exchange rate regimes (see appendix). In the remainder of this paper, the fixed exchange rate version of the model is used. Causal influence graphs of the flexible exchange rate regime can be found in Berndsen (1992).

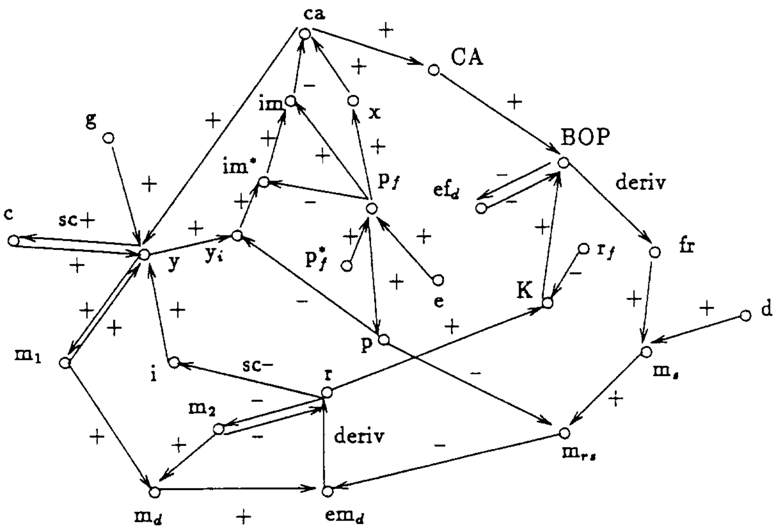

Fig. 1. The constraint graph of the Mundell–Fleming model.

consists of a single variable and all left-hand sides are different [Boutillier (1984)]:

$$y_i = g_i(y, z), \qquad i = 1, \ldots, n. \tag{1}$$

Equation i represents the determination mechanism of endogenous variable y_i, i.e., the left-hand side variable is said to be *determined by* the variables on the right-hand side. Given this notion of 'determined by' in economic equations, we have defined the same notion in the context of constraints in table 1. In each type of constraint in table 1, w is the 'left-hand side' variable and the other variables are on the 'right-hand side'. Hence, in each elementary constraint graph corresponding to a type of constraint, the links are directed from the other variables to w.

Analogous to the causal ordering approach of Gilli (1984), we assume the existence of a perfect matching from right-hand-side variables to left-hand-side variables. In other words, it is assumed that every endogenous variable appears exactly once as variable w in the set of constraints. However, we prefer the matching defined by table 1 to other possible matchings. This particular matching reflects the economic dependencies embedded in the set of constraints. Consider the set of constraints $\mathscr{C}$ with, on the one hand, the SC- and DERIV-constraints and, on the other hand, the SUM-constraints.

The semantics of the SC- and DERIV-constraints leads naturally to a preference of directing the links in one direction. In the case of the SC-constraints, the direction of the link is according to time, i.e., from lagged variables to variables referring to the current period. The direction of the link between two nodes corresponding to the pair of variables occurring in a DERIV-constraint, is obvious from the economic interpretation of the DERIV-constraint. Such links point from the variable representing excess demand on market i to the price variable of market i.

SUM-constraints represent definitional equations, accounting identities, and behavioural equations consisting of contemporaneous variables. Firstly, in definitional equations, one of the variables, say A, is defined in terms of other variables. It is assumed that the reason for incorporating this relation in the first place is, that A can be explained on the basis of the other variables [see, e.g., Boutillier (1984)]. Otherwise, the definition can be dropped from the model without loss of information. Therefore, we assume that A appears in the position of variable w in the SUM-constraint of table 1. Secondly, accounting identities represent the total on one side of the balance which is the sum of balance sheet items. We argue that it is more natural to reason from the parts to the whole than vice versa. Therefore, the direction of the links originating from balance sheet equations is from the parts to the whole. Finally, in behavioural equations consisting of contemporaneous variables, the causal dependencies among variables should be derived from the underlying economic theory. If variable A 'depends on' the set of variables $B_1, \ldots, B_n$, then the constraint $\mathrm{SUM}(A, (s_1 B_1), \ldots, (s_n B_n))$ is appropriate, where s_i represents the sign of the partial derivative of B_i.

It is obvious that table 1 defines an explicit representation of causality. The constraint graph is the graphical representation of the causal dependencies already embedded in the constraints. Therefore, in order to derive a causal ordering which agrees with 'economic intuition', it is important that the set of constraints is modelled on the basis of economic knowledge.

In the following, we consider the process of generating sequences of admissible qualitative states by means of state transitions. In principle, all admissible states of $\mathscr{S}$ can be determined given a set of variables $\mathscr{V}$, quantity spaces $\mathscr{Q}$, and constraints $\mathscr{C}$. A *valid state transition* is an ordered pair $QS(\mathscr{V}, t_k), QS(\mathscr{V}, t_{k+1})$ of admissible states such that for all $v \in \mathscr{V}$, $QS(v, t_k), QS(v, t_{k+1})$ is a valid variable transition. The set of *valid variable transitions* consists of two disjoint subsets QD and QS as shown in table 2 [see also Berndsen and Daniels (1990)]. The set of QD-transitions is relevant for variable v if $QSval(v) = \{?\}$. Otherwise, $QSval(v) = \{-, 0, +\}$ and so-called QS-transitions apply to v. These tables specify all ordered pairs of qualitative states corresponding to valid variable transitions, i.e., any ordered pair of qualitative states which is not in table 2, is not a valid variable transition.

A *qualitative behaviour* of a variable v from t_k to t_{k+n} is a sequence of qualitative states with valid state transitions between them:

$$QS(v, t_k), \ldots, QS(v, t_{k+n}).$$

Table 2

QD-transitions and QS-transitions.

	$Qdir(v, t_k)$	→ $Qdir(v, t_{k+1})$		$QS(v, t_k)$	→ $QS(v, t_{k+1})$
QD_1	Any[a]	std	QS_1	$(0, std)$	$(0, Any)$
QD_2	Any	inc	QS_2	$(0, inc)$	$(+, Any)$
QD_3	Any	dec	QS_3	$(0, dec)$	$(-, Any)$
			QS_4	$(+, dec)$	$(0, Any)$
			QS_5	$(+, Any)$	$(+, Any)$
			QS_6	$(+, dec)$	$(-, Any)$
			QS_7	$(-, inc)$	$(0, Any)$
			QS_8	$(-, Any)$	$(-, Any)$
			QS_9	$(-, inc)$	$(+, Any)$

[a] QD_i is a subset of three transitions with $Any \in Qdir$; analogously, QS_i is a subset of nine transitions ($i = 5, 8$) or three transitions (otherwise).

Accordingly, a qualitative behaviour of the system $\mathcal{S}$ from t_k to t_{k+n} is the corresponding sequence of admissible qualitative states of $\mathcal{S}$.

Given an initial state $QS(\mathcal{V}, t_1)$, we can determine the set of admissible states which are reachable from $QS(\mathcal{V}, t_1)$ by valid state transitions. Usually, the initial state $QS(\mathcal{V}, t_1)$ represents the situation immediately after a disturbance of the equilibrium position; the disturbance is represented by an exogenous variable v with $Qdir(v) = inc$ or $Qdir(v) = dec$. Following De Kleer and Brown (1984), the process of generating the set of admissible states is called 'envisioning'. The result of this simulation is represented as a directed graph called the complete envisionment.

Definition 2. The complete envisionment *of* $\mathcal{S}$ *with initial state* $QS(\mathcal{V}, t_1)$ *is a rooted directed graph E with the following properties:*

- $QS(\mathcal{V}, t_1)$ *is the root.*
- *The set of nodes of E contains all admissible qualitative states of* $\mathcal{S}$ *that are reachable from the root by valid state transitions.*
- *There is a link between two nodes of E iff there exists a valid state transition between them.*

A path from the root to another node in the envisionment corresponds to some qualitative behaviour of the economic system. The envisionment is the description of all possible qualitative behaviours of the model starting at the root. In the case of the Mundell–Fleming model the envisionment contains over 2000 different qualitative states. The number of qualitative behaviours is therefore intractably large. Many of these behaviours do not have a causal explanation. They only differ in small changes in cycles or repetitions of states. In the following sections, we develop a method for pruning the envisionment such that only those economic behaviours remain which have a causal explanation. This method is based on the so-called causality heuristic described in section 4.

3. Causal influence graphs

In this section, we describe a method which derives a causal ordering of variables in constraint graphs. The application of this method to constraint graph G yields a so-called causal influence graph (CIG) which represents the causal structure of the model. Henceforth, we assume that $\mathscr{S}$ satisfies the following two conditions. First, the system is self-contained, i.e., $|\mathscr{V}| = |\mathscr{C}|$. Second, every variable $v \in \mathscr{V}$ appears exactly once on the position of variable w in constraint $c \in \mathscr{C}$ as defined in table 1. These assumptions are quite natural in economics. Since, if the number of variables exceeds the number of constraints, arbitrary choices have to be made which 'endogenous' variables are in fact exogenous. Conversely, if the number of constraints is greater than the number of variables, the constraint graph is not a directed graph.

Let $R(V)$ denote the reachable set of nodes V, i.e., the set of points reachable from some node of V. The strong component containing node v is denoted by $S(v)$. The *condensation* D^* of a digraph D is a digraph with the set of k strong components $S_1, \ldots, S_k$ of D as nodes. There is a link in D^* from $S(v)$ to $S(w)$ iff $S(v)$ and $S(w)$ are distinct, and there is a link from v to w in D. A subgraph D_U of *D generated by the vertex set U* is the subgraph with vertex set U containing all links of D that join two points of U. In addition, we define the *predecessor set* $P(v)$ as the set of nodes with a link pointing to v. $\mathrm{OUT}(v)$ is defined as the set $\{u \notin S(v) \mid v \in R(u)\}$, i.e., $\mathrm{OUT}(v)$ is the set of nodes with a path to v outside the strong component containing v. The subset of nodes $u \in \mathrm{OUT}(v)$ with a shortest path to v is defined by

$$\mathrm{OUT}_{md}(v) = \{u \in \mathrm{OUT}(v) \mid \forall_{w \in \mathrm{OUT}(v)}\, d(w, v) \geq d(u, v)\}\,.$$

Hence, $d(u, v)$ is the minimal distance, denoted by $md(v)$, from any node $u \in \mathrm{OUT}(v)$ to v. Finally, a strong component $S(v)$ consisting of a single vertex v is called a *singleton* strong component.

Given the constraint graph G we can construct the causal influence graph by applying a node numbering algorithm which orders vertices of G. For clarity, we construct the intermediate graphs which are employed consecutively to derive the causal influence graph.

Firstly, given the constraint graph G, all links in G originating from SC-constraints are removed. In case of a static model, the resulting graph coincides with G. Secondly, the condensation graph is constructed. For example, the condensation graph of the Mundell–Fleming model is depicted in fig. 2. This graph is equivalent to the causal graph which would be obtained by applying the program CAUSOR [Gilli (1984)].

Thirdly, a source set (V_0) is chosen, i.e., a subset of the set of singleton strong components in the condensation graph. The reasoning about change of the variables starts at variables in the source set since these are not causally

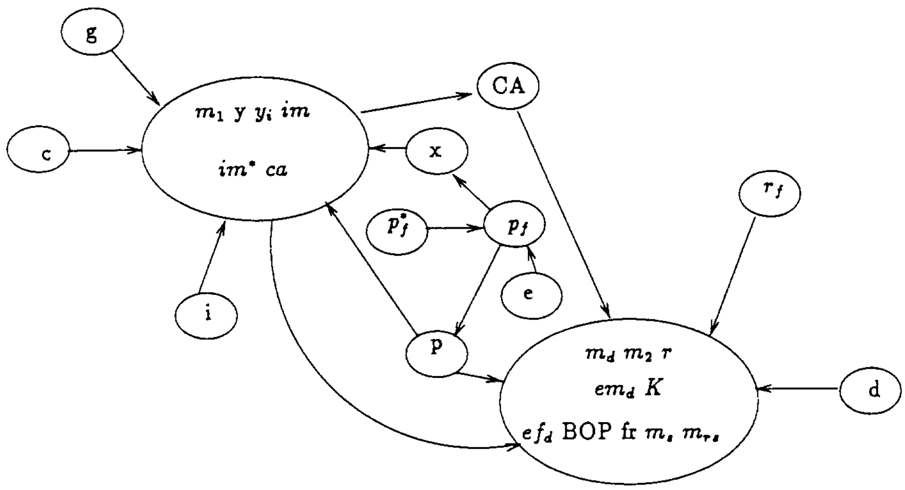

Fig. 2. The condensation graph of the Mundell–Fleming model.

dependent on variables in the same time point. The source set V_0 can be of type 1 or 2. The source set is of type 1 if it consists of one or more exogenous variables. V_0 is of type 2 if it contains variables which depend on lagged variables. For example, in the condensation graph of the Mundell–Fleming model $\{g, r_f, e\}$ and $\{c, i\}$ are respectively source sets of type 1 and 2.

Let L denote the set of nodes $V_0 \cup R(V_0)$. Let G_L denote the subgraph of the condensation graph generated by L. In other words, G_L is the maximal subgraph of the condensation graph in which all nodes are removed which are not in the source set or reachable from the source set. Then, the numbered constraint graph is derived from G_L by labelling the nodes of G_L with numbers as follows:

Definition 3 (*Numbered constraint graph*). *Given the source set V_0, the numbered constraint graph (NG_L) is derived from G_L by numbering the nodes such that:*

(1) *For* $v \in V_0$, $n(v) = 0$.

(2) *For* $v \in R(V_0)$, $n(v) = \mathrm{Max}\{n(w) \mid w \in \mathrm{OUT}_{md}(v)\} + md(v)$.

Note that for $v \in R(V_0)$, $\mathrm{OUT}_{md}(v)$ is nonempty. Furthermore, all variables within a strong component are numbered. The node numbering algorithm which yields a numbered constraint graph NG_L can be found in Berndsen (1992). The graph NG_L for the Mundell–Fleming model and source set $V_0 = \{g\}$ is shown in fig. 3.

The numbering of the nodes in NG_L leads naturally to the notion of antecedent set. A vertex v is said to be antecedent to vertex w iff there exists a link vw in NG_L and $n(v) < n(w)$. The *antecedent set* of w, denoted $\mathrm{ANT}(w)$, is the set of all

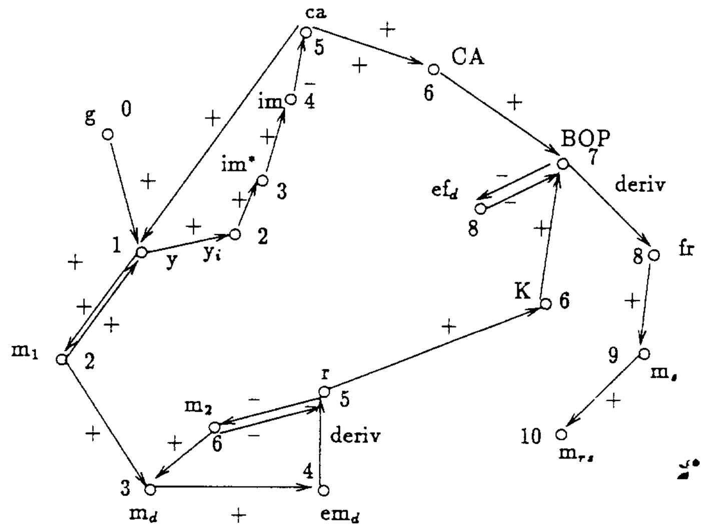

Fig. 3. The graph NG_L of the Mundell–Fleming model with $V_0 = \{g\}$.

variables corresponding to vertices v that are antecedent to w. Obviously, the antecedent set is empty if w is a source variable. A link vw in NG_L is defined as a *nonantecedent* link iff $n(v) \geq n(w)$.

Finally, the *causal influence graph* is the subgraph of NG_L obtained by deleting all nonantecedent links. The causal influence graph is of type 1 or 2 depending on the type of the source set V_0. The causal influence graph CIG(g) of the Mundell–Fleming model is depicted in fig. 4.

In order to show the economic relevance of the causal influence graph, we confront the causal explanation obtained from this graph with the explanation given by Fleming (1962). The issue at hand is to explain the consequences of an increase in public expenditure. The explanation by Fleming is quoted below in italics in two parts. To facilitate the comparison, the corresponding parts of the reasoning along the edges of the CIG have been inserted between square brackets in the quotation. In addition, we show the increase (↑) or decrease (↓) of each variable. The changes of each variable are derived from the qualitative state $QS(\mathscr{V}, t_1)$ shown in table 3.[3] The formal derivation of this qualitative state is outlined in the next section.

Under fixed exchange rates, an increase in public expenditure will give rise to an increase in income which will be associated – if the economy was previously

[3] In the corresponding causal envisionment there are three qualitative states at t_1 which differ only with respect to *Qdir*(*BOP*). Since *Qdir*(*CA*) = *dec*, *Qdir*(*K*) = *inc*, and SUM(*BOP*, + *CA*, + *K*), it follows that *Qdir*(*BOP*) = ?.

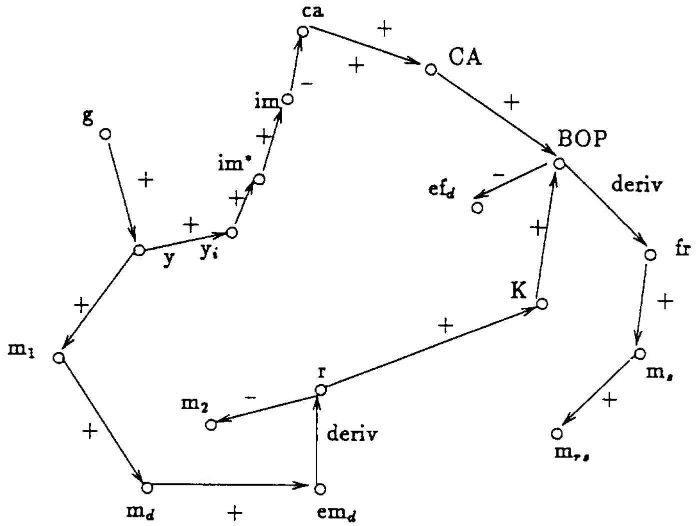

Fig. 4. The causal influence graph CIG(g) of the Mundell–Fleming model.

underemployed – with increases in employment and output [$g \uparrow \xrightarrow{+} y \uparrow$]. *The increase in expenditure will lead to a deterioration in the balance of payments on current account, owing, notably, to a rise in imports* [$y \uparrow \xrightarrow{+} y_i \uparrow \xrightarrow{+} im^* \uparrow \xrightarrow{+} im \uparrow \xrightarrow{-} ca \downarrow \xrightarrow{+} CA \downarrow \xrightarrow{+} BOP \downarrow$]... *Since the increase in public expenditure provokes an unfavorable shift in the current balance and a favorable shift in the capital balance* [$y \uparrow \xrightarrow{+} m_1 \uparrow \xrightarrow{+} m_d \uparrow \xrightarrow{+} em_d \uparrow \xrightarrow{\text{deriv}} r \uparrow \xrightarrow{+} K \uparrow \xrightarrow{+} BOP \uparrow$], *it is uncertain whether the balance of payments as a whole will deteriorate or improve.*

From fig. 4, it is clear that a change in g affects BOP in two ways. It is not possible to conclude from CIG(g) which of the two influences dominates the other. In the second part of the quotation, Fleming discusses some conditions under which the unfavourable effect ($CA \downarrow \xrightarrow{+} BOP \downarrow$) outweighs the favourable effect ($K \uparrow \xrightarrow{+} BOP \uparrow$). Again, the corresponding links are inserted between square brackets in the quotation:

...It is the more likely to deteriorate, and the less likely to improve, the higher is the marginal propensity to import [$y \xrightarrow{+} im$], *the less sensitive is the rate of interest to changes in money income* [$em_d \xrightarrow{\text{deriv}} r$], *and the less sensitive are capital movements to changes in the rate of interest* [$r \xrightarrow{+} K$].

From this comparison, we may conclude that the causal dependencies in the Mundell–Fleming model, quoted from Fleming (1962, p. 370), are similar to the

Table 3

The causal state of the Mundell–Fleming model after an increase in government expenditures.

v	$QS(v, t_1)$	v	$QS(v, t_1)$
BOP	$-, ?$	m_d	?, *inc*
c	?, *inc*	m_1	?, *inc*
ca	?, *dec*	m_2	?, *dec*
CA	?, *dec*	m_{rs}	?, *dec*
d	?, *std*	m_s	?, *dec*
e	?, *std*	p_d	?, *std*
ef_d	?, *inc*	p	?, *std*
em_d	+, *inc*	p_f	?, *std*
fr	?, *dec*	p_f^*	?, *std*
g	?, *inc*	r	?, *inc*
i	?, *std*	r_f	?, *std*
im	?, *inc*	x	?, *std*
im^*	?, *inc*	y	?, *inc*
K	?, *inc*	y_i	?, *inc*

causal dependencies obtained from the CIG in fig. 4. Moreover, from table 3, it is clear that the changes of the variables of the model, indicated by ↑ and ↓ above, are also similar. The causal influence graph leads to the formulation of the causality heuristic. The kind of reasoning presented above is a sloppy way of applying the causality heuristic described more formally in the next section. In fact, the reasoning presented in Fleming (1962) only describes the initial disturbance in qualitative terms.

4. The causality heuristic

The causality heuristic filters admissible states which are incompatible with the causal structure of the economic system as represented by the CIG. It reduces ambiguities since the influence of nodes which are not antecedent to another node are not considered. Thus, the causality heuristic follows the line of reasoning represented in the CIG. We define a notion of compatibility of an admissible state $QS(\mathcal{V}, t)$ with the causal influence graph CIG_i.

Definition 4 (antecedent compatible). The admissible qualitative state $QS(w, t)$ of variable w at time t is antecedent compatible iff w is a node of the CIG and belongs to one of the following mutually exclusive cases:

(1) *w is a source variable.*

(2) *$ANT(w) = \{v_1, \ldots, v_k\}$ and the tuple $(QS(w, t), QS(v_1, t), \ldots, QS(v_k, t))$ satisfies the conditions as defined in table 4, where $l_{v_i w}$ denotes the label of the link between the node corresponding to v_i and the node corresponding to w.*

Table 4
Antecedent compatible states.

$ANT(w)$	$l_{v_i w}$	$(QS(w, t), QS(v_1, t), \dots, QS(v_k, t))$
$\{v_1\}$	$+$	$QS(w, t) = QS(v_1, t)$
$\{v_1\}$	$-$	$QS(w, t) = \ominus QS(v_1, t)$
$\{v_1\}$	deriv	$\text{DERIV}(w, v_1)$
$\{v_1, \dots, v_k\}$	$s_1, \dots, s_k$	$QS(w, t) \approx s_1 QS(v_1, t) \oplus, \dots, \oplus s_k QS(v_k, t)$

Thus, if w is a source variable then $QS(w, t)$ is antecedent compatible by definition. Otherwise, there are two mutually exclusive cases to consider. If the antecedent set consists of a single variable, then this is the only variable which influences w, and hence the label l_{vw} determines the kind of influence of $QS(v, t)$ with respect to $QS(w, t)$. Otherwise, the antecedent set contains more than one variable. In that case, there are multiple influences on w. Each label $l_{v_i w}$, considered in isolation, indicates the influence v_i would exert on w if it were the only predecessor of w in the CIG. The joint influence is determined by the sum of the partial influences. Therefore, the qualitative state of w is antecedent compatible if $QS(w, t)$ is qualitatively equal to the joint influence as defined in table 4.

Given the definition of antecedent compatibility, we define a causal state as follows: $QS(\mathscr{V}, t)$ is a causal state iff $QS(\mathscr{V}, t)$ is an admissible state, and for all v corresponding to a node in the CIG, $QS(v, t)$ is antecedent compatible.

We define the application of the *causality heuristic* to a causal state S as follows. Given the set A of admissible successors of S, discard all noncausal states of A and retain all causal states. Given an initial state S_0, the recursive application of the causality heuristic to S_0 and its successors yields the so-called causal envisionment. Hence the causality heuristic removes all noncausal states and thus yields a subgraph of the complete envisionment.

In case of the Mundell–Fleming model, the *complete* envisionment contains more than 2000 nodes. The *causal* envisionment consists of only 75 nodes and captures the behaviours for which an explanation can be given in terms of causal dependencies as embodied by the causal influence graph. This envisionment can be pruned further if assumptions about the 'strength' of links in the CIG are made. In Berndsen (1992), we have derived general upper bounds for the number of states in the envisionment. Let $N(E)$, $N(E_c)$, and $N(E_{Mc})$ denote, respectively, the number of nodes in the complete envisionment, the causal envisionment, and the OM-causal envisionment,[4] and let $\overline{N(E)}$, $\overline{N(E_c)}$, and $\overline{N(E_{OMc})}$ denote the

[4]This envisionment is generated by reasoning mode 5 which combines the causality heuristic with a kind of order-of-magnitude heuristic.

Table 5
Comparison of upper bounds and actual values.

Model	$\overline{N(E)}$	$N(E)$	$\overline{N(E_c)}$	$N(E_c)$	$\overline{N(E_{OMc})}$	$N(E_{OMc})$
Mundell–Fleming	2187	2081	243	75	81	39

theoretical upper bounds of these quantities. The results in case of the Mundell–Fleming model are presented in table 5. From this table, it is clear that the application of heuristics can reduce the envisionment considerably.

5. QERT

The computer program QERT (Qualitative Economic Reasoning Tool) is a qualitative reasoner implemented in Prolog which can be used to generate the envisionment of an economic system. In addition, various reasoning modes may be used in order to select a subgraph of the complete envisionment. In fig. 5, the input and output of the program are shown schematically.

The *input* of QERT, independent of the reasoning mode, consists of the initial state $QS(\mathcal{V}, t_1)$ and the economic system $\mathcal{S}$. The initial state is defined by the following term: *node*(1, *InitialState*, [*root*], *t*(1), [ ]). By definition, the initial state has number $N = 1$ and time interval t_1. Furthermore, the list of predecessors is empty because it is the root of the graph. The economic system is represented in QERT by the following predicate:

$$economic\_system(Name,\ Variables,\ QuantitySpaces,\ Constraints)\,.$$

where *Name* is the name of the economic system, *Variables* is the list of variables, *QuantitySpaces* is the list of quantity spaces $QSval_j$ for each variable v_j, and

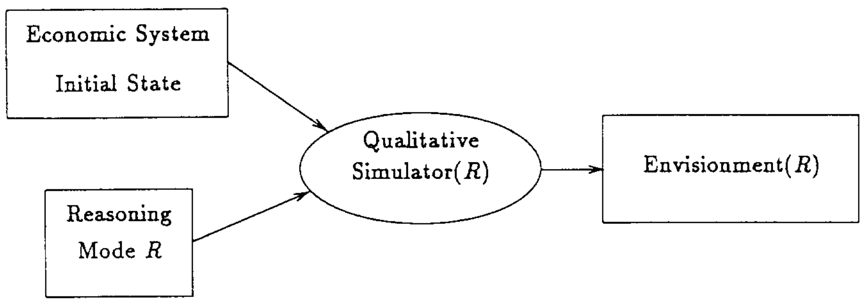

Fig. 5. Input and output of QERT.

Constraints is the list of constraints. Each constraint is represented by the term *c*(*Cname*, *Variables*), where *Cname* is one of the following labels {add, sum, deriv, sc, mplus, mmin, exo} and *Variables* is the list of variables.

The 'reasoning mode' denotes the kind of envisionment generated by QERT. It is specified by the user in the clause mode_of_reasoning(Mode). QERT offers five modes for generating the envisionment of an economic system:

R	Envisionment(R)
1	Complete envisionment
2	Equilibrium behaviour by applying the equilibrium heuristic $h_e(n)$
3	OM-envisionment
4	Causal envisionment
5	Combination of 3 and 4

In this paper, we discussed the fourth reasoning mode in particular. For a detailed description of the other reasoning modes, see Berndsen (1992).

Each of the five reasoning modes generates an envisionment which is a rooted directed graph. The envisionment is represented by instances of the following clause:

node(*N*, *QualitativeState*, *Label*, *Time*, *Predecessors*),

where N is a number to identify the node uniquely, *QualitativeState* is a list of qualitative states of the variables in $\mathscr{V}$, *Time* denotes the time interval, *Label* is a label to identify equilibrium and no-change nodes, and *Predecessors* is a list of immediate predecessor nodes of N.

6. Conclusions

In this paper we have developed techniques for qualitative reasoning in economic systems. It has been shown that qualitative economic reasoning can be formalised to the extent that qualitative behaviour and associated explanations can be obtained that correspond to economic reasoning put forward by economists. The basic framework in section 2 is capable of generating all possible behaviours of an economic system but of which many have no satisfactory economic explanation. The causal dependencies of the system play a crucial role in selecting those behaviours which are meaningful from the economic point of view. These causal dependencies can be represented in the form of a causal influence graph which in turn forms the basis of the causality heuristic. This heuristic serves as a filter to obtain feasible behaviours for which a causal explanation can be generated. All procedures have been tested on a simple Keynesian model and the Mundell–Fleming model using the computer program QERT implemented in Quintus Prolog.

Appendix: The Mundell–Fleming model

Here we present the Mundell–Fleming model referred to in this paper. The model consists of three markets: the goods market, the money market, and the foreign exchange market, listed respectively in tables 6, 7, and 8. On the right-hand side of these tables the equations of the model are shown. The corresponding constraints are listed on the left-hand side. The equations which depend on the exchange rate regime are listed in table 8. The sign s_i of variable v_i in a SUM- or SC-constraint is derived from the sign of the partial derivative of v_i in the corresponding equation. The list of variables is presented at the end of the appendix.

By inspection of tables 6, 7, and 8, it is easy to show that this set of constraints meets the conditions for deriving causal influence graphs set out in section 3.

Table 6

The goods market.

SUM(y, + c, + i, + g, + ca)	$y = c + i + g + ca$
SUM(y_i, + y, + p_d, − p)	$y_i = f_1(y, p_d, p)$
SUM(p, + p_d, + p_f)	$p = \gamma p_d + (1-\gamma)p_f$
SUM(p_f, + p_f^*, + e)	$p_f = f_2(p_f^*, e)$
SC(c, + y)	$c = f_3(y_{t-1})$
SC(i, − r)	$i = f_4(r_{t-1})$
SUM(ca, + x, − im)	$ca = x - im$
SUM(x, + p_f, − p_d)	$x = f_5(p_f, p_d)$
SUM(im, + im^*, + p_f, − p_d)	$im = f_6(im^*, p_f, p_d)$
SUM(im^*, + y_i, − p_f, + p_d)	$im^* = f_7(y_i, p_f, p_d)$
EXO(g)	
EXO(p_d)	
EXO(p_f^*)	

Table 7

The money market.

SUM(m_d, + m_1, + m_2)	$m_d = m_1 + m_2$
$M^+(m_1, y)$	$m_1 = f_8(y)$
$M^-(m_2, r)$	$m_2 = f_9(r)$
SUM(m_{rs}, + m_s, − p)	$m_{rs} = f_{10}(m_s, p)$
SUM(em_d, + m_d, − m_{rs})	$em_d = m_d - m_{rs}$
DERIV(r, em_d)	$\dot{r} = f_{11}(em_d)$
SUM(m_s, + fr, + d)	$m_s = fr + d$
EXO(d)	

Table 8
The foreign exchange market.

SUM(BOP, + CA, + K)	$BOP = CA + K$
SUM(CA, + p_d, + ca)	$CA = f_{12}(p_d, ca)$
SUM(K, + r, − r_f)	$K = f_{13}(r, r_f)$
$M^-(ef_d, BOP)$	$ef_d = -BOP$
EXO(r_f)	
Fixed exchange rate regime	
DERIV(fr, BOP)	$\dot{fr} = f_{14}(BOP)$
EXO(e)	
Flexible exchange rate regime	
DERIV(e, ef_d)	$\dot{e} = f_{15}(ef_d)$
EXO(fr)	

This follows from the fact that the model is self-contained (28 variables and 28 constraints) and that every variable appears exactly once on the position of variable w in a constraint $c \in \mathscr{C}$ as defined in table 1.

List of variables

BOP	Balance of payments
c	Real private consumption
ca	Current account of BOP
CA	Current account of BOP in domestic currency units
d	Domestic assets of central bank
e	Exchange rate
ef_d	Excess demand for foreign exchange
em_d	Excess demand for money
fr	Foreign exchange reserves
g	Real government expenditures
i	Real private investment
im	Real imports in terms of domestic goods
im^*	Real imports in terms of foreign goods
K	Capital account of BOP
m_d	Total money demand
m_1	Transactions money demand
m_2	Speculative money demand
m_{rs}	Real money supply

m_s	Money supply
p_d	Price of domestic good (in domestic currency units)
p	Average price of domestic and foreign goods
p_f	Price of foreign good (in domestic currency units)
p_f^*	Price of foreign good (in foreign currency units)
r	Interest rate
r_f	Foreign interest rate
x	Real exports in terms of domestic goods
y	National income in terms of domestic goods
y_i	National income

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